Code : 303102

BCA 1st Semester Exam., 2023

BASIC MATHEMATICS

Time : 3 hours *Full Marks* : 60

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **SEVEN** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question Nos. **1 & 2** are compulsory.*

1. Choose the correct answer of the following (any *six*) : 2×6=12

(a) The number of significant digits in the number 204·020050 is

(i) 5

(ii) 6

(iii) 8

(iv) 9

(b) Let R be a non-empty relation defined on a collection of sets as ARB if and only if $A \cap B = \phi$. Then

(i) R is reflexive and transitive

(ii) R is symmetric and not transitive

(iii) R is an equivalence relation

(iv) R is not reflexive and not symmetric

(c) Let A be a finite set of size n. The number of elements in the power set of $A \times A$ is

(i) 2^{2^n}

(ii) 2^{n^2}

(iii) $(2^n)^2$

(iv) $(2^2)^n$

(d) Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is

(i) n

(ii) n^2

(iii) 1

(iv) $(n+1)$

(e) Let E, F and G be finite sets. Let

$$X = (E \cap F) - (F \cap G)$$
$$Y = (E - (E \cap G)) - (E - F)$$

Which one of the following is true?

(i) $X \subset Y$

(ii) $X \supset Y$

(iii) $X = Y$

(iv) $X - Y \neq \varphi$ and $Y - X \neq \varphi$

(f) In a set of people, the relation x is not older than y satisfies which property?

(i) Transitivity

(ii) Reflexivity

(iii) Antisymmetric

(iv) Symmetric

(g) A survey shows that 63% of Indians like Punjabi food, whereas 76% like South Indian food. If x% of Indians like both Punjabi and South Indian meals, then

(i) $23 \leq x \leq 63$

(ii) $23 \leq x \leq 69$

(iii) $39 \leq x \leq 69$

(iv) $39 \leq x \leq 63$

(h) The propositional statement

$$(P \to (Q \vee R)) \to ((P \wedge Q) \to R$$

is

(i) satisfiable but not valid

(ii) valid

(iii) a contradiction

(iv) None of the above

(i) Which of the following relations on $\{1,2,3\} \to \{5,6,7,8\}$ is an injective function?

(i) $1 \to 6, 2 \to 7, 3 \to 5$

(ii) $1 \to 7, 2 \to 7, 3 \to 5$

(iii) $1 \to 8, 2 \to 5, 3 \to 8$

(iv) $1 \to 6, 2 \to 7, 3 \to 5, 1 \to 8$

(Turn Over)

(j) Let S be a set of n elements. The numbers of ordered pairs in the largest and the smallest equivalence relations on S are

(i) n and n

(ii) n^2 and n

(iii) n^2 and 0

(iv) n and 1

2. Answer any *three* of the following : 4×3=12

(a) Justify the following proposition is tautology or not :

$$P \wedge (\sim P \vee Q)$$

(b) Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. Simplify the expression

$$(A - B) \cup (B - A) \cup (A \cap B)$$

(c) Give an example of a relation R which is symmetric and transitive but not reflexive.

(d) Out of a group of 21 persons, 9 eat vegetables, 10 eat fish, 7 eat eggs, and 5 persons eat all three. How many persons eat at least two out of the three dishes?

(e) Let R_1 and R_2 be two equivalence relations on a set. Is $R_1 \cup R_2$ an equivalence relation? Justify.

3. Find the value of the integral

$$\int_0^{\frac{\pi}{4}} (\tan x + \cot x)^{-2} dx$$ 12

4. If $y = x^3 e^{ax}$, then find y_n, using Leibnitz theorem. 12

5. If $x = t^2 - 1$ and $y = 2e^t$, then find d^2y / dx^2. 12

6. Find the number of subsets of $\{1, 2, \ldots, n\}$ with odd cardinality. 12

7. How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set? 12

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